Toroidal dipole resonances in the relativistic random phase approximation

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The isoscalar toroidal dipole strength distributions in spherical nuclei are calculated in the framework of a fully consistent relativistic random phase approximation, based on effective mean-field Lagrangians with nonlinear meson self-interaction terms. It is suggested that the recently observed "low-lying component of the isoscalar dipole mode" might in fact correspond to the toroidal giant dipole resonance. Although predicted by several theoretical models, the existence of toroidal resonances has not yet been confirmed in experiment. In the present analysis the vortex dynamics of these states is displayed by the corresponding velocity fields.

In addition to data on isoscalar giant monopole resonances (IS GMR) in spherical nuclei, it is expected that information on nuclear matter incompressibility should also be obtained in studies of the isoscalar dipole mode. The isoscalar giant dipole resonance (IS GDR) is a second order effect, built on $3\hbar\omega$, or higher configurations. It corresponds to a compression wave traveling back and forth through the nucleus along a definite direction. Recent data on IS GDR obtained by using inelastic scattering of α particles on ^{208}Pb [1], and on ^{90}Zr, ^{116}Sn, ^{144}Sm, and ^{208}Pb [2], have been analyzed in the non-relativistic Hartree-Fock plus RPA framework [3], and with relativistic mean-field plus RPA (RRPA) calculations [4]. Both analyses have shown that: (a) there is a strong disagreement between theory and the reported experimental data on the position of the IS GDR centroid energies, and (b) calculations predict the splitting of the IS GDR strength distribution into two broad structures, one in the the high-energy region above 20 MeV, and one in the low-energy window between 8 MeV and 14 MeV. Effective interactions, both non-relativistic and relativistic, which reproduce the experimental excitation energies of the IS GMR, predict centroid energies of the IS GDR that are $4-5$ MeV higher than those extracted from small angle α-scattering spectra. Not only is this disagreement between theory and experiment an order of magnitude larger than for other giant resonances, but also the present data on IS GDR are not consistent with the value of the nuclear incompressibility K_A derived from the measured excitation energy of the isoscalar GMR [5]. Another puzzling result is the theoretical prediction of a substantial amount of isoscalar dipole strength in the $8-14$ MeV region. In Ref. [4] we have shown that the RRPA peaks in this region do not correspond to a compression mode, but rather to a kind of toroidal motion with dynamics determined by surface effects. In a very recent article [6], Clark *et al.* reported new experimental data on the isoscalar dipole strength functions in ^{90}Zr, ^{116}Sn, and ^{208}Pb, measured with inelastic scattering of α particles at small angles. They found that the isoscalar E1 strength distribution in each nucleus consists of a broad component at $E_x \approx 114/A^{1/3}$ MeV containing approximately 100% of the E1 EWSR, and a narrower one at $E_x \approx 72/A^{1/3}$ MeV containing $15-28$ % of the total isoscalar E1 strength. The higher component is identified as the E1 compression mode, whereas the lower component may be the new mode predicted by the RRPA analysis of Ref. [4]. In the present work we suggest that the observed low-lying E1 strength may correspond to the toroidal giant dipole resonance (TGDR).

The role of toroidal multipole form factors and moments in the physics of electromagnetic and weak interactions has been extensively discussed in Refs. [7] and [8]. They appear in multipole expansions for systems containing convection and induction currents. In particular, the multipole expansion of a four-current distribution gives rise to three families of multipole moments: charge moments, magnetic moments and electric transverse moments. The later are related to the toroidal multipole moments and result from the expansion of the transverse electric part of the current. The toroidal dipole moment, in particular, describes a system of poloidal currents on a torus. Since the charge density is zero for this configuration, and all the turns of the torus have magnetic moments lying in the symmetry plane, both the charge and magnetic dipole moments of this configuration are equal to zero. The simplest model is an ordinary solenoid bent into a torus.

Vortex waves in nuclei were analyzed in a hydrodynamic model [9]. By relaxing the assumption of irrotational motion, in this pioneering study solenoidal toroidal vibrations were predicted, which correspond to the toroidal giant dipole resonance at excitation energy $E_x \approx (50-70)/A^{1/3}$ MeV. It was suggested that the vortex excitation modes should appear in electron backscattering. In the framework of the time-dependent Hartree-Fock theory, the isoscalar 1^- toroidal dipole states were also studied by analyzing the dynamics of the moments of the Wigner transform of the density matrix [10].

In this Letter the toroidal dipole strength distributions are calculated in the relativistic random phase approximation (RRPA). The RRPA represents the small amplitude limit of the time-dependent relativistic mean-field theory [11]. A self-consistent calculations ensures that the same correlations which define the ground-state properties, also determine the behavior of small deviations from the equilibrium. The same effective Lagrangian generates the Dirac-Hartree single-particle spectrum and the residual particle-hole interaction. In a number of recent applications [4,12–16] it has been shown that, by using effective Lagrangians which in the mean-field approximation provide an accurate description of ground-state properties, excellent agreement with experimental data is also found for the excitation energies of low-lying collective states and of giant resonances. Two points are essential for the successful application of the RRPA in the description of dynamical properties of finite nuclei: (i) the use of effective Lagrangians with non-linear terms in the meson sector, and (ii) the fully consistent treatment of the Dirac sea of negative energy states. In particular, in Ref. [15] it has been shown that configurations which include negative-energy states have an especially pronounced effect on isoscalar excitation modes.

In Fig. 1 we display the RRPA toroidal dipole strength distributions in ^{90}Zr, ^{116}Sn, and ^{208}Pb:

$$R(E) = \sum_i B^{T=0}(E1, 1_i^- \to 0_f) \frac{\Gamma^2}{4(E-E_i)^2 - \Gamma^2}, \tag{1}$$

where Γ is the width of the Lorentzian distribution, and

$$B^{T=0}(E1, 1_i^- \to 0_f) = \frac{1}{3} |\langle 0_f || \hat{T}_1^{T=0} || 1_i^- \rangle|^2. \tag{2}$$

For the strength distributions in Fig. 1 the width is $\Gamma = 1.0$ MeV. The isoscalar toroidal dipole operator is defined [7]

$$\hat{T}_{1\mu}^{T=0} = -\sqrt{\pi} \int \left[r^2 \left(\overrightarrow{Y}^*_{10\mu} + \frac{\sqrt{2}}{5} \overrightarrow{Y}^*_{12\mu} \right) - < r^2 >_0 \overrightarrow{Y}^*_{10\mu} \right] \cdot \vec{J}(\vec{r})\, d^3r. \tag{3}$$

In the relativistic framework the expression for the the isoscalar baryon current reads

$$J^\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \psi_i, \tag{4}$$

where the summation is over all occupied states in the Fermi sea. The resulting toroidal dipole operator is

$$\hat{T}_{1\mu}^{T=0} = -\sqrt{\pi} \sum_{i=1}^{A} \left[r_i^2 \left(\overrightarrow{Y}^*_{10\mu}(\Omega_i) + \frac{\sqrt{2}}{5} \overrightarrow{Y}^*_{12\mu}(\Omega_i) \right) \cdot \vec{\alpha}_i - < r^2 >_0 \overrightarrow{Y}^*_{10\mu}(\Omega_i) \cdot \vec{\alpha}_i \right], \tag{5}$$

where $\overrightarrow{Y}_{ll'\mu}$ denotes a vector spherical harmonic, and $\vec{\alpha}$ are the Dirac α-matrices. The calculations have been performed with the self-consistent Dirac-Hartree plus relativistic RPA. The effective mean-field Lagrangian contains nonlinear meson self-interaction terms, and the configuration space includes both particle-hole pairs, and pairs formed from hole states and negative-energy states. The inclusion of the term $- < r^2 >_0 \overrightarrow{Y}^*_{10\mu}$ in the toroidal operator ensures that the TGDR strength distributions do not contain spurious components that correspond to the center-of-mass motion [9].

The strength distributions in Fig. 1 have been calculated with the NL1 [17], NL3 [18] and NL-SH [19] effective interactions. These three forces, in order of increasing values of the nuclear matter compressibility modulus, have been extensively used in the description of a variety of properties of finite nuclei. In particular, in Ref. [20] it has been shown that the NL3 ($K_{\rm nm} = 271.8$ MeV) effective interaction provides the best description of experimental data on isoscalar giant monopole resonances. We note that the toroidal dipole strength is indeed concentrated in the energy region where "the low-lying component of the IS GDR" has been observed [6]. The toroidal strength distributions in all three nuclei are fragmented into two broad structures, and their positions should be compared with the experimental centroid energies of the "low-lying component" [6]: 16.2 ± 0.8 MeV for ^{90}Zr, 14.7 ± 0.5 MeV for ^{116}Sn, and 12.2 ± 0.6 MeV for ^{208}Pb. There is no toroidal strength calculated above $\approx$ 20 MeV, and the strength functions in Fig. 1 display only a weak dependence on the nuclear matter incompressibility of the effective forces. Note that, for example in the case of ^{208}Pb, the toroidal strength distribution is found exactly in the same energy region in which the "low-lying IS GDR component" has been calculated (see Fig. 1 of Ref. [4]). On the other hand, no toroidal strength is found in the region of the compression mode, i.e. the "high-lying IS GDR" above 20 MeV.

The dynamics of the solenoidal toroidal vibrations is illustrated in Fig. 2, where we display the velocity field for the most pronounced dipole peak at 18.43 MeV in ^{116}Sn, calculated with the NL3 effective interaction. The velocity distributions are derived from the corresponding transition currents, following the procedure described in Ref. [21]. A vector of unit length is assigned to the largest velocity. All the other velocity vectors are normalized accordingly. Since the collective flow is axially symmetric, we plot the velocity field in cylindrical coordinates. The z-axis corresponds to the symmetry axis of a torus. The velocity field in the $(z, r_\perp)$ plane corresponds to poloidal currents on a torus with vanishing inner radius. The poloidal currents determine the dynamical toroidal moment.

The coupling between the toroidal mode and the isoscalar dipole compression mode becomes evident if one rewrites the expression in square brackets of the toroidal operator (3) as [9]

$$\nabla \times (\vec{r} \times \nabla)(r^3 - \frac{5}{3} < r^2 >_0 r)\ Y_{1\mu}, \tag{6}$$

and compares it with the isoscalar dipole operator of the compression mode [4]

$$\hat{Q}_{1\mu}^{T=0} = \sum_{i=1}^{A} \gamma_0\,(r^3 - \frac{5}{3} < r^2 >_0 r)\ Y_{1\mu}(\theta_i, \varphi_i). \tag{7}$$

The mixing between the two modes is also illustrated in Fig. 3, where we plot the velocity fields for the four most pronounced peaks of the toroidal dipole strength distributions in ^{208}Pb, calculated with the NL3 effective interaction (see Fig. 1). The lowest peak at 9.54 MeV contains strong contributions of the dipole compression mode which, for the NL3 interaction, is calculated at $\approx$ 26 MeV (compare with Fig. 3 of Ref. [4]). The "squeezing" compression mode is identified by the flow lines which concentrate in the two "poles" on the symmetry axis. The velocity field corresponds to a density distribution which is being compressed in the lower half plane, and expands in the upper half plane. The centers of compression and expansion are located on the symmetry axis, at approximately half the distance between the center and the surface of the nucleus. The compression mode admixtures are also seen in the weak dependence of the toroidal strength distributions on the nuclear matter incompressibility of the effective forces (see Fig. 1). The higher peaks, and especially the two at 14.49 MeV and 15.72 Mev, are completely dominated by vortex collective motion. With increasing energy the center of the vortex moves toward the surface of the nucleus. For the peak at 14.49 MeV the center of the vortex is at 3.7 fm, while it is found at 4.2 fm for the 15.72 MeV toroidal state.

In conclusion, we suggest that the recently observed "low-lying component of the isoscalar dipole mode" [6] might in fact correspond to the toroidal giant dipole resonance. By employing the fully consistent relativistic random phase approximation, we have shown that the toroidal dipole strength is indeed concentrated in the energy region in which the low-lying component of the IS GDR is calculated. The vortex nature of the collective motion is displayed in the analysis of the velocity fields. In general, a pronounced coupling of the two dipole modes, toroidal and compressional, is expected in the low energy region. On the other hand, no toroidal strength is found in the high-energy region above 20 MeV, where the compression mode is located.

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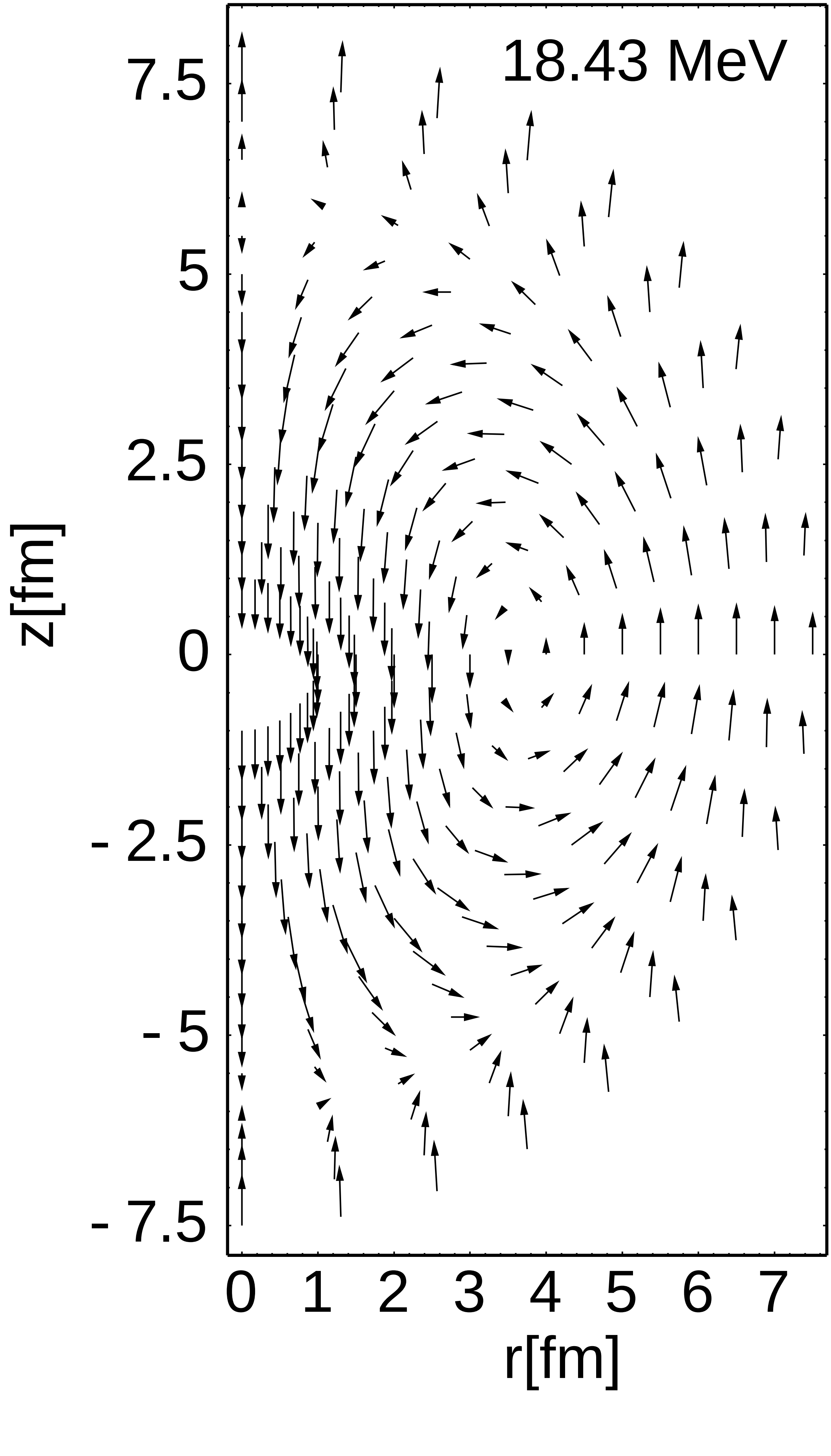
FIG. 1. Toroidal dipole strength distributions in ^{90}Zr, ^{116}Sn and ^{208}Pb, calculated with the NL1 (dashed), NL3 (solid), and NL-SH (dot-dashed) effective interactions.

FIG. 2. Velocity distribution for the toroidal dipole peak at 18.43 MeV in ^{116}Sn, calculated with the NL3 effective interaction.

FIG. 3. Velocity distributions for the toroidal dipole mode in ^{208}Pb, calculated with the NL3 effective interaction. The velocity fields correspond to the peaks at 9.54 MeV (a), 10.48 MeV (b), 14.49 MeV (c), and 15.72 Mev (d).


18.43 MeV
7.5
5
2.5
0
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z[fm]
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6
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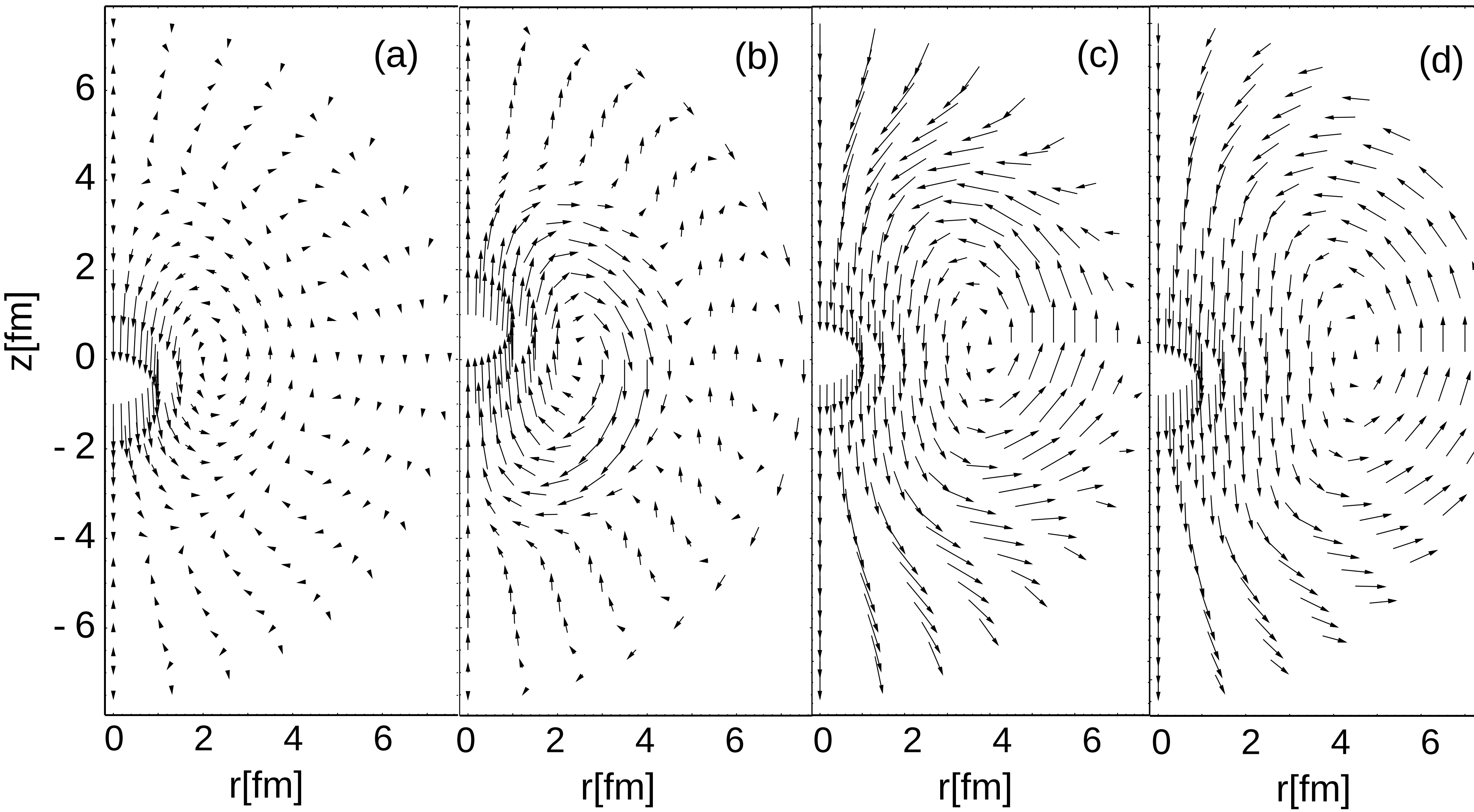
(a)
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z[fm]
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0
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